Some inverse eigenvalue problems for Jacobi matrices

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Abstract

Let

$$T_{1,n} = \begin{pmatrix} \alpha_1 & \beta_1 & & & \\ \beta_1 & \alpha_2 & \ddots & & 0 \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \beta_{n-1} \\ 0 & & & \beta_{n-1} & \alpha_n \end{pmatrix}.$$

Denote

$$T_{p,q} = \begin{pmatrix} \alpha_p & \beta_p & & & 0 \\ \beta_p & \alpha_{p+1} & \beta_{p+1} & & \\ & \beta_{p+1} & \ddots & \ddots & \\ & & \ddots & \ddots & \beta_{q-1} \\ 0 & & & \beta_{q-1} & \alpha_q \end{pmatrix} (p < q \leq n.)$$

If all $\beta_i > 0 \quad i = 1, 2, ..., n-1$,we call $T_{1,n}$ a Jacobi matrix

The following 3 kinds of inverse eigenvalue problem for Jacobi matrices will be discussed

1.(K)problem[1],[2]:Given 3 sets of real numbers $\{\lambda_1, \lambda_2, ..., \lambda_n\}$, $\{\mu_1, \mu_2, ..., \mu_{k-1}\}$, $\{\mu_k, \mu_{k+1}, ..., \mu_{n-1}\}$, find a $n \times n$ Jacobi matrix $T_{1,n}$,such that $\lambda_1, \lambda_2, ..., \lambda_n$ are eigenvalues of $T_{1,n}$,$\mu_1, \mu_2, ..., \mu_{k-1}$ are eigenvalues of $T_{1,k-1}$ and $\mu_k, \mu_{k+1}, ..., \mu_{n-1}$ are the eigenvalues of $T_{k+1,n}$.

2.Double dimension problem[3][4][5][6]:given a Jacobi matrix $T_{1,n}$and given 2n real numbers $\{\lambda_1, \lambda_2, ..., \lambda_{2n}\}$, find a $2n \times 2n$ Jacobi matrix $T_{1,2n}$, such that $T_{1,n}$ is a leading principal submatrix of $T_{1,2n}$ and $\lambda_1, \lambda_2, ..., \lambda_{2n}$ are the eigenvalues of $T_{1,2n}$.

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A periodic Jacobi matrix is an $n \times n$ real symmetric matrix of the form

$$J_n = \begin{pmatrix} \alpha_1 & \beta_1 & & & & \beta_n \\ \beta_1 & \alpha_2 & \beta_2 & & & 0 \\ 0 & \beta_2 & \alpha_3 & \ddots & & 0 \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ 0 & & & \ddots & \alpha_{n-1} & \beta_{n-1} \\ \beta_n & 0 & \cdots & 0 & \beta_{n-1} & \alpha_n \end{pmatrix}.$$

where $\beta_i > 0, \quad i = 1, 2, ..., n.$

3.Periodic problem [4][5],[7]:Given two sets of real numbers $\{\lambda_1, \lambda_2, ..., \lambda_n\}$ and $\{\mu_1, \mu_2, ..., \mu_{n-1}\}$,find a $n \times n$ periodic Jacobi matrix J_n,such that $\lambda_1, \lambda_2, ..., \lambda_n$ are the eigenvalues of J_n and $\mu_1, \mu_2, ..., \mu_{n-1}$ are the eigenvalues of $T_{1,n-1}$ which is the $(n-1) \times (n-1)$leading principal submatix of J_n

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