

Basic notions of Vector Space Theory: students' models and conceptions

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Carlson (1993) uses the image of the fog rolling in to describe the confusion and disorientation which his students experience when getting to the basic notions of Vector Space Theory (VST). There is truly a widespread sense of the inadequacy of the teaching of Linear Algebra. On account of that common perception and of the importance of Linear Algebra as a prerequisite for a number of disciplines (math, science, engineering,...), in the last twenty years several studies were carried out on Linear Algebra education. Those studies brought undeniable progresses for understanding students difficulties in Linear Algebra. As Dorier and Sierpinska effectively synthesized in their literature survey (2001), three different kinds of sources of students difficulties in Linear Algebra especially emerge from the studies on that topics:

1. the fact that Linear Algebra teaching is characterized by an axiomatic approach, which is perceived by students as superfluous and meaningless;
2. the fact that Linear Algebra is characterized by the cohabitation of different languages, systems of representations, modes of description;
3. the fact that coping with those features requires the development of *theoretical thinking* and *cognitive flexibility*

Recently more studies were carried out, which in our opinion still fit well Dorier and Sierpinska's synthesis.

In this talk I will focus on some aspects of students' difficulties in vector space theory (VST), drawn from my doctorate research project. That project was meant to investigate graduate and undergraduate students errors and difficulties in VST. Through that work I intended to contribute to Linear Algebra Education research field, focusing on cognitive difficulties related to specific VST notions rather than to general features of Linear Algebra: a seemingly less explored path.

The study involved 15 (graduate or undergraduate) students in mathematics, presented with two or three different VST problems to be solved in individual sessions. The methodology adopted was that of the clinical interview (Ginsburg, 1981). The study highlighted a number of students' difficulties related

to the notions of linear combination, linear dependence/independence, dimension and spanning set. The difficulties, errors and impasses emerged were analysed through the lenses of different theoretical frameworks: the theory of tacit intuitive models (Fischbein, 1987), Sfard's process-object duality theory (Sfard, 1991) and the ckc model (Balacheff, 1995). The different analyses lead to formulate hypotheses, which account for a variety of students' difficulties. Though not antithetical to each other, those analyses are diversified, put into evidence different aspects from different perspectives. In this talk I briefly present the results of those analyses and a first tentative integrating analysis, combining different hints and perspectives provided by the frameworks mentioned above. More specifically, that attempt led to the formulation of the hypothesis that many difficulties experienced by students are consistent with the possible activation of an intuitive model of construction related to basic notion of VST. In the talk we will better specify that hypothesis showing how it could contribute to better organize and explain students' documented difficulties.

References

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