1 Numerical ranges of nilpotent operators

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For any operator A on a Hilbert space, let $w(A)$ and $w_0(A)$ denote its numerical radius and the distance from the origin to the boundary of its numerical range, respectively. We prove that if A is nilpotent with nilpotency n, then $w(A)$ is at most the product of $n-1$ and $w_0(A)$. When A attains its numerical radius, we also determine a necessary and sufficient condition for the equality to hold.