1 Convex and Non-convex Optimization Problems for the Field of Values of a Matrix

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We introduce and study numerical algorithms that compute the minimal and maximal distances between $0 \in \mathbb{C}$ and points in the field of values $F(A) = \{x^*Ax \mid x \in \mathbb{C}^n\ ,\ \|x\|_2 = 1\} \subset \mathbb{C}$ for a complex matrix $A_{n,n}$. Finding the minimal distance from $0 \in \mathbb{C}$ to $F(A)$ is a convex optimization problem if $0 \notin F(A)$ and thus it has a unique solution, called the Crawford number whose magnitude relates information on the stability margin of the associated system. If $0 \in F(A)$, this is a non-convex optimization problem and consequently there can be multiple solutions or local minima that are not so globally. Non-convexity also holds for the maximal distance problem between points in $F(A)$ and $0 \in \mathbb{C}$. This maximal distance is commonly called the numerical radius $numrad(A)$ for which the inequality $\rho(A) \leq numrad(A) \leq \|A\|$ is well established.
Both cases can be solved efficiently numerically by using ideas from geometric computing, eigenanalyses of linear combinations of the hermitean and skew-hermitean parts of A and the rotation method introduced by C. R. Johnson in the 1970s to compute the boundary of the field of values.