Spectral Manifolds

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It is well known that the set of all $n \times n$ symmetric matrices of rank k is a smooth manifold. This set can be described as those symmetric matrices whose ordered vector of eigenvalues has exactly $n - k$ zeros. The set of all vectors in $\mathbb{R}^n$ with exactly $n - k$ zero entries is itself an analytic manifold. In this work, we characterize the manifolds M in $\mathbb{R}^n$ with the property that the set of all $n \times n$ symmetric matrices whose ordered vector of eigenvalues belongs to M is a manifold. In particular, we show that if M is a C^2, C^∞, or C^ω manifold then so is the corresponding matrix set. We give a formula for the dimension of the matrix manifold in terms of the dimension of M.