On the problem of diagonalizability of matrices over a principal ideal domain

By *Volodymyr Prokip*.

Let R – be a principal ideal domain with the unit element $e \neq 0$ and $U(R)$ the set of divisors of unit element e. Further, let R_n – the ring of $(n \times n)$-matrices over R; I_k – the identity $k \times k$ matrix and O the zero $n \times n$ matrix. In this report we present conditions of diagonalizability of a matrix $A \in R_n$, i.e. when for A there exists a matrix $T \in GL(n, R)$ such that TAT^{-1} – a diagonal matrix.

Theorem. Let $A \in R_n$ and

$$\det(Ix - A) = (x - \alpha_1)^{k_1}(x - \alpha_2)^{k_2} \cdots (x - \alpha_r)^{k_r},$$

where $\alpha_i \in R$, and $\alpha_i - \alpha_j \in U(R)$ for all $i \neq j$. If $m(x) = (x - \alpha_1)(x - \alpha_2) \cdots (x - \alpha_r)$ – the minimal polynomial of the matrix A, i.e. $m(A) = O$, then for the matrix A there exists a matrix $T \in GL(n, R)$ such that

$$TAT^{-1} = \operatorname{diag}\left(\alpha_1 I_{k_1}, \alpha_2 I_{k_2}, \ldots, \alpha_r I_{k_r}\right).$$