1 Tensor Sylvester matrices and information matrices of multiple stationary processes

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Consider the matrix polynomials $A(z)$ and $B(z)$ given by

$$A(z) = \sum_{j=0}^{p} A_j z^j$$

and

$$B(z) = \sum_{j=0}^{q} B_j z^j,$$

where $A_0 \equiv B_0 \equiv I_n$.
Gohberg and Lerer [1] study the resultant property of the tensor Sylvester matrix $\mathcal{S}^{\otimes}(-B, A) \triangleq \mathcal{S}(-B \otimes I_n, I_n \otimes A)$ or $\mathcal{S}^{\otimes}(-B, A) =$

$$\left(\begin{array}{ccc} (-I_n) \otimes I_n & (-B_1) \otimes I_n & \cdots \\ 0_{n^2 \times n^2} & \ddots & \ddots \\ \vdots & \ddots & \ddots \\ 0_{n^2 \times n^2} & \cdots & 0_{n^2 \times n^2} \\ I_n \otimes I_n & I_n \otimes A_1 & \cdots \\ 0_{n^2 \times n^2} & \ddots & \ddots \\ \vdots & \ddots & \ddots \\ 0_{n^2 \times n^2} & \cdots & 0_{n^2 \times n^2} \end{array} \right.$$

In [1] it is proved that the matrix polynomials $A(z)$ and $B(z)$ have at least one common eigenvalue if and only if $\det \mathcal{S}^{\otimes}(-B, A) = 0$ or when the matrix $\mathcal{S}^{\otimes}(-B, A)$ is singular. In other words, the tensor Sylvester matrix $\mathcal{S}^{\otimes}(-B, A)$ becomes singular if and only if the scalar polynomials det $A(z) = 0$ and det $B(z) = 0$ have at least one common root. Consequently, it is a multiple resultant. In [2], this property is extended to the Fisher information matrix of a stationary vector autoregressive and moving average process, VARMA process. The purpose of this talk consists of displaying a representation of the Fisher information matrix of a stationary VARMAX process in terms of tensor Sylvester matrices, the X stands for exogenous or control variable. The VARMAX process is of common use in stochastic systems and control.