From Total Positivity to Positivity: related classes of matrices

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Matrices with all their minors nonnegative (respectively, positive) are usually called totally nonnegative (respectively, totally positive). These matrices present nice stability properties as well as interesting spectral, factorization and variation diminishing properties. They play an important role in many applications to other fields such as Approximation Theory, Mechanichs, Economy, Optimization, Combinatorics or Computer Aided Geometric Design. We revisit some of the properties and applications of these matrices and show some recent advances. Moreover, we show that some results and techniques coming from Total Positivity theory have been extended to other classes of matrices which are also closely related to positivity. Among these other classes of matrices we consider sign regular matrices (which generalize totally nonnegative matrices), some classes of P-matrices (matrices whose principal minors are positive), including M-matrices, and conditionally positive definite (and conditionally negative definite) matrices.