Structured Hölder condition numbers for eigenvalues under fully nongeneric perturbations

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Let λ be an eigenvalue of a matrix or operator A. The condition number $\kappa(A, \lambda)$ measures the sensitivity of λ with respect to arbitrary perturbations in A. If A belongs to some relevant class, say $\mathbb{S}$, of structured operators, one can define the *structured* condition number $\kappa(A, \lambda; \mathbb{S})$, which measures the sensitivity of λ to perturbations *within* the set $\mathbb{S}$. Whenever the structured condition number is much smaller than the unstructured one, thepossibility opens for a structure-preserving spectral algorithm to be more accurate than a conventional one.

For multiple, possibly defective, eigenvalues the condition number is usually defined as a pair of nonnegative numbers, with the first component reflecting the worst-case asymptotic order which is to be expected from the perturbations in the eigenvalue. In this talk we adress the case when this asymptotic order differs for structured and for unstructured perturbations: if we denote $\kappa(A, \lambda) = (n, \alpha)$ and $\kappa(A, \lambda; \mathbb{S}) = (n_{\mathbb{S}}, \alpha_{\mathbb{S}})$, we consider the case when $n \neq n_{\mathbb{S}}$, i.e., when structured perturbations induce a *qualitatively* different perturbation behavior than unstructured ones. If this happens, we say that the class $\mathbb{S}$ of perturbations is *fully nongeneric* for λ.

On one hand, full nongenericity is characterized in terms of the eigenvector matrices corresponding to λ, and it is shown that, for linear structures, this is related to the so-called skew-structure associated with $\mathbb{S}$. On the other hand, we make use of Newton polygon techniques to obtain explicit formulas for structured condition numbers in the fully nongeneric case: both the asymptotic order and the largest possible leading coefficient are identified in the asymptotic expansion of perturbed eigenvalues for fully nongeneric perturbations.