1 Applications of generalized inverses in art.

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The Moore-Penrose inverse of a matrix A corresponds to the (unique) matrix solution X of the system AXA=A, XAX=X, (AX)*=AX, (XA)*=XA. S. L. Campbell and C. D. Meyer Jr, wrote a now classical book Generalized Inverses of Linear Transformations (Pitman Publishing Limited, London, 1979), in which they gave an excellent account on the MP-inverse and other generalized inverses as well. They gave many interesting examples, ranging from Gauss historical prediction for finding Ceres to modern electrical engineering problems. The present paper provides new applications related to art studies: a first one about mathematical colour theory, and a second about curve fitting in architectural drawings or paintings. Firstly, in colour theory, a frequent problem is finding the combination of colours approximating a desired colour as closely as possible using a given set of colours. Plaid fabrics are made by a limited number of threads and when a desired tone cannot be formed by a combination, a least squares approach may be mandatory. Some colour theory specialists suggested sensations, such as the observation of colour, should involve logarithmic functions, but using Campbell and Meyers general set-up, this does not give rise to additional difficulties. Of course, the practical use of this theory should still show the benefit of the proposed mathematical tool, but even as stands it already provides a colourful mathematical diversion. In addition, colour theory as taught today in many art schools and as used in numerous printing or computer problems, is in need of a more rigid mathematical approach, for sure. Thus, this example of an application of the theory of general inverses in art may be welcomed. Secondly, we turn to the formerly very popular activity in architectural circles of drawing all kinds of geometric figures on images of artworks and buildings. Until some 20 years ago, triangles, rectangles, pentagons or circles sufficed, but later more general mathematical figures were used as well, especially since fractals became trendy. Recognizing well-known curves and polygons was seen as a part of the interpretation of an architectural edifice or painting. Eventually, certain proportions in the geometric figures were emphasized, among which the golden section surely was the most (in)famous. Diehards continue this tradition, though curve drawing has lost some credit in recent times, in particular due to some exaggerated golden section interpretations. Today, many journals tend to reject geometric readings in architecture, and the reasons to do so are many. For instance, an architect may have had the intention of constructing a certain curve, but for structural, technical or whatever practical reason, the final realization may not confirm that intention. Or else, a certain proportion may have been used in an artwork, consciously or not, but when such a hidden proportion is discovered afterward, even the author of the artwork may disagree on having used it. Consequently, statements about the presence of a certain proportion or about the good fit of a curve in art often are subjective matters, and thus unacceptable for scientific journals. However, a similarity between these geometric studies in architecture and the history of (celestial) mechanics, as explained Generalized Inverses of Linear Transformations, suggests the so-called least squares method, developed in that field, could be applied to examples in art as well. Just as astronomy struggled for centuries to get rid of its astrological past, an objective approach for the described art studies would be most welcome. Of course, it can be opposed the mathematical method presents an overkill with respect to the intended straightforward artistic applications, but nowadays software considerably reduces the computational aspects. The method turns out to be useful indeed: for instance, while a catenary approximates architect Gaudis Paelle Guell better than a parabola, the least squares method shows a catenary or a parabola can be used for the shape of Gaudis Collegio Teresiano with a comparable error. These results were confirmed by Prof. A. Monreal, a Gaudi specialist from the architects hometown, Barcelona. Another amusing example is the profile of a nuclear power plant, which is described in many schoolbooks as an example of a hyperbola, but an ellipse fits even better. Engineers confirmed the hyperbolic shape is modified at the top to reduce wind resistance. Finally, it is shown how proportions in the Mona Lisa can be studied using generalized inverses, but it remains unsure this application will make the present paper as widely read as Dan Browns da Vinci Code.