1 Commutativity preserving maps on real matrices

By *Ajda Fošner*.

Let $M_n(\mathbb{R})$ be the algebra of all $n \times n$ real matrices. A map $\phi : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ preserves commutativity if $\phi(A)\phi(B) = \phi(B)\phi(A)$ whenever $AB = BA$, $A, B \in M_n(\mathbb{R})$. If ϕ is bijective and both ϕ and ϕ^{-1} preserve commutativity, then we say that ϕ preserves commutativity in both directions. We will talk about non-linear maps on $M_n(\mathbb{R})$ that preserve commutativity in both directions or in one direction only.