1 On the permuted max-algebraic eigenvector problem

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Let $a \oplus b = \max(a, b)$, $a \otimes b = a + b$ for $a, b \in \overline{\mathbb{R}} := \mathbb{R} \cup \{-\infty\}$ and extend these operations to matrices and vectors as in conventional linear algebra. The following *max-algebraic eigenvector problem* has been intensively studied in the past: Given $A \in \overline{\mathbb{R}}^{n\times n}$, find all $x \in \overline{\mathbb{R}}^{n}, x \neq (-\infty, ..., -\infty)^T$ (*eigenvectors*) such that $A \otimes x = \lambda \otimes x$ for some $\lambda \in \overline{\mathbb{R}}$. In our talk we deal with the *permuted eigenvector problem*: Given $A \in \overline{\mathbb{R}}^{n\times n}$ and $x \in \overline{\mathbb{R}}^{n}$, is it possible to permute the components of x so that the arising vector x' is a (max-algebraic) eigenvector of A? This problem can be proved to be NP-complete using a polynomial transformation from BANDWIDTH. As a by-product the following *permuted max-linear system problem* can also be shown NP -complete: Given $A \in \overline{\mathbb{R}}^{m\times n}$ and $b \in \overline{\mathbb{R}}^{m}$, is it possible to permute the components of b so that for the arising vector b' the system $A \otimes x = b'$ has a solution? Both problems can be solved in polynomial time when n does not exceed 3.