

Toeplitz matrices with Fisher-Hartwig symbols

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Asymptotic properties of large Toeplitz matrices are best understood if the matrix is constituted by the Fourier coefficients of a smooth function without zeros on the unit circle and with winding number zero. If at least one of these conditions on the generating function is violated, one speaks of Toeplitz matrices with Fisher-Hartwig symbols. The talk is intended as an introduction to the realm of Toeplitz matrices with Fisher-Hartwig symbols for a broad audience. We show that several highly interesting and therefore very popular Toeplitz matrices are just matrices with a Fisher-Hartwig symbol and that many questions on general Toeplitz matrices, for example, the asymptotics of the extremal eigenvalues, are nothing but specific problems for matrices with Fisher-Hartwig symbols. We embark on both classical and recent results concerning the asymptotic behavior of determinants, condition numbers, eigenvalues, and eigenvectors as the matrix dimension goes to infinity.