1 Model Reduction for unstable Systems based on Hierarchical Matrix Arithmetic

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We consider linear time-invariant (LTI) systems of the following form

$$\Sigma : \begin{cases} \dot{x}(t) &= Ax(t) + Bu(t), \qquad t > 0, \qquad x(0) = x^0, \\ y(t) &= Cx(t) + Du(t), \qquad t \geq 0, \end{cases}$$

with $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times m}$, and $C \in \mathbf{R}^{p\times n}$ arising, e.g., from the discretization and linearization of parabolic PDEs. We will assume that the system Σ is large-scale with $n \gg m$, p and that the system is unstable, satisfying

$$\Lambda(A) \cap \mathbf{C}^{+} \neq \emptyset, \quad \Lambda(A) \cap \jmath\mathbf{R} = \emptyset.$$

We further allow the system matrix A to be dense, provided that a *data-sparse* representation exists. To reduce the dimension of the system Σ, we apply an approach based on the controllability and observability Gramians of Σ. The numerical solution of these Gramians is obtained by solving two algebraic Bernoulli and two Lyapunov equations. As standard methods for the solution of matrix equations are of limited use for large-scale systems, we investigate approaches based on the *matrix sign function* method. To make this iterative method applicable in the large-scale setting, we incorporate structural information from the underlying PDE model into the approach. By using data-sparse matrix approximations, hierarchical matrix formats, and the corresponding formatted arithmetic we obtain an efficient solver having linear-polylogarithmic complexity. Once the Gramians are computed, a reduced-order system can be obtained applying the usual *balanced truncation method.*